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Probabilistic Crack Growth Model for Application to Composite Solid Propellants

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In this study a crack growth model, based on probabilistic mechanics, is developed. The developed crack growth model yields a power law relationship between the crack growth rate and the mode I stress intensity factor. The sensitivity of the crack growth rate to the variation of the parameters in the crack growth model is investigated and the results are discussed. The limitations and the applicability of using the developed crack growth model to predict crack growth behavior are also discussed.

Introduction

IN past years, a considerable amount of work has been done in studying crack growth behavior in solid propellants[1–5] based on linear viscoelastic and elastic fracture mechanics theories. The basic approach used in these studies is to relate the crack growth rate to the stress intensity factor. Experimental data indicate that a power law relationship exists between the crack growth rate and the stress intensity factor. This experimental finding supports the theory developed by Knauss[6] and Schapery[7] in their studies of crack growth behavior in linear viscoelastic materials. The existence of a good correlation between the crack growth rate and the stress intensity factor implies that the crack growth behavior is controlled by the local stress near the crack tip. Therefore, it is expected that the local microstructure will have a significant effect on the crack growth behavior.

It is well known that a highly filled composite solid propellant, on the microscopic scale, can be considered as nonhomogeneous material. Depending on the degree of cross linking of the matrix material, filler particle size and distribution, and the bond strength at the interface of the particle and the matrix, the local stress and strength will vary in a random fashion. Therefore, it is reasonable to expect that the failure location ahead of the crack tip will also vary in a random manner. In other words, failure may not occur at the location where the local stress attains a highest value. In addition, since the failure of the material is closely related to the damage state and since the damage process is a time-dependent process, it is expected that the failure time will also vary randomly. From the probabilistic viewpoint, the failure site and the associated failure time should be considered as random variables. Therefore, to obtain a fundamental understanding of the crack growth behavior in solid propellants, it is desirable to develop a crack growth model based on probabilistic mechanics.

Various stochastic crack growth models have been proposed in the literature, mainly for metallic materials and super-alloys.[8–20] However, all of the stochastic crack growth models proposed, with the exception of Ref. 20, are based on $\mathrm{d}a/\mathrm{d}n = XF(\Delta K, R, a)$ in which $F(\Delta K, R)$ is a well-known deterministic crack growth rate function that is a function of the stress intensity range ΔK, the stress ratio R and the crack size a, and X is a random process. X can be a random process of time, $X(t)$ (Refs. 11–13), a random process of the stress intensity range, $X(\Delta k)$ (Ref. 10), or a random process of the crack size, $X(a)$ (Ref. 18). The random process X can be a continuous diffusion Markov process[11,12] or a random variable.[10,13,17,18] The approaches just described are based on the randomization of well-known crack growth rate functions that are used for the deterministic crack propagation analysis. To date, no stochastic crack growth model for composite solid propellants has been proposed in the literature.

In this study, a probabilistic crack growth model is developed for composite solid propellants. Instead of randomizing a deterministic crack growth rate function, we investigate the probabilistic crack growth behavior ahead of the crack tip. The stochastic crack growth model thus established provides relations between crack growth rate parameters and physical material properties. The developed model indicates that a power law relationship exists between the crack growth rate, $\mathrm{d}a/\mathrm{d}t$, and the mode I stress intensity factor K_I.

It is shown that the average crack growth rate depends on the failure process zone size W, the parameters α and β of the Weibull distribution for the strength of the material element, and the constants b and g of the damage function for a material element ahead of the crack tip. The sensitivity of the crack growth rate to the variation of the aforementioned parameters is investigated and the results discussed. In addition, the limitations and the applicability of using the developed crack growth model to predict crack growth behavior are also discussed.

Crack Propagation Model

In developing the crack growth model in this study, the material ahead of the crack tip is divided into a number of independent elements. The elements in a small region, known as the failure process zone, in the immediate vicinity of the crack tip, are assumed to be damaged according to a damage distribution inside the failure process zone. However, the elements outside the failure process zone are assumed to be undamaged. These assumptions are approximately consistent with the experimental and analytical results reported by Liu[21–23] in his study of damage evolution in a composite solid propellant. It is further assumed that the strength of each material element is statistically independent and follows the Weibull distribution.[24] This assumption was verified by analyzing the strength data obtained by conducting constant strain rate tests on minidogbone specimens.

The crack propagation process consists of a series of discrete events, in which a crack growth increment, Δa occurs suddenly at discrete times. Both the magnitude of the crack growth increment Δa and the duration between two crack growth events ΔT are random variables. The event of crack increment occurs when the strength of a material element at a distance Δa ahead of the crack tip is exceeded by the accumulated damage in the time period ΔT. In other words, the crack size $a(t)$ remains constant for a time

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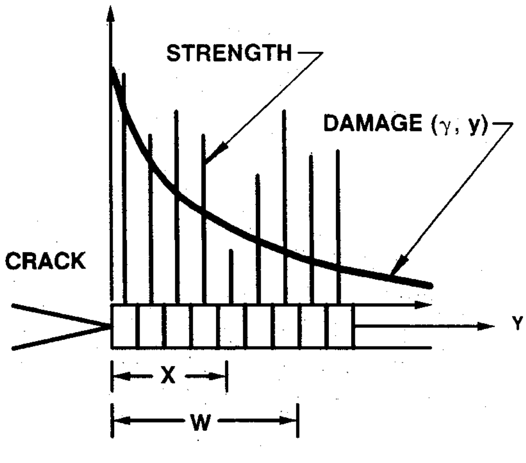

Fig. 1 Crack growth model.

period of ΔT and then jumps by an amount of Δa. Of course, both random variables ΔT and Δa depend on the crack size $a(t)$ and the applied stress at time t. The crack growth rate, denoted by $\dot{a}(t)$, is given by

$$\dot{a}(t) = \Delta a / \Delta T \tag{1}$$

The event of crack increment occurs when the strength of a small element at a distance Δa ahead of the crack tip is exceeded by the accumulated applied damage in the time period ΔT, as shown in Fig. 1.

Let $R(y)$ be the strength of a small element at a distance y from the crack tip. It is a random variable assumed to follow the Weibull distribution[24]

$$F_{R(y)}(r) = P[R(y) \le r] = \frac{1}{\nu}\left(\frac{r-\epsilon}{\beta}\right)^{a} \Delta y; \qquad r \ge \epsilon$$

$$= 0; \qquad r < \epsilon \tag{2}$$

in which α and β are the shape parameter and the scale parameter, respectively, and ϵ is the lower bound of the strength. In Eq. (2), ν is a unit length so that $F_{R(y)}(r)$ is nondimensional.

It is assumed that the strength of each small element ahead of the crack tip is statistically independent and identically distributed.[24] In other words, $R(y_1)$ is statistically independent of $R(y_2)$ and for $y_1 \neq y_2$, and the distributions of $R(y_1)$ and $R(y_2)$ are identical as given by Eq. (2). This assumption will be discussed further subsequently.

Based on the crack propagation mechanism described earlier, the joint probability density function of random variables ΔT and Δa, denoted by $f_{\Delta T \Delta a}(\tau, x)$, can be expressed in the following:

$$\begin{aligned} f_{\Delta T\Delta\alpha}(\tau, x)\,\mathrm{d}r\,\mathrm{d}x &= P[\tau < \Delta T \le \tau + \Delta\tau,\ x < \Delta a \le x + \Delta x] \\ &= P[R(y) > D(\tau, y),\ \forall y < x] \\ &\times P[D(\tau, x) < R(x) \le D(\tau + \Delta\tau, x)] \\ &\times P[R(y) > D(\tau + \Delta\tau, y),\ \forall y > x] \end{aligned} \tag{3}$$

in which $D(\tau, y)$ is the accumulated damage during the time period τ in an element at a distance y from the crack tip.

On the right-hand side of Eq. (3), the first term denotes the probability that all of the elements at a distance $y < x$ survive for a period of time τ, the second term indicates the probability that the element at x fails in the time interval $(\tau, \tau + \Delta\tau)$, and the third term represents the probability that all of the elements at a distance greater than x from the crack tip survive for a period time $\tau + \Delta\tau$. The probabilities mentioned are multiplied together because of the assumption that the strength of each small material element is statistically independent.

Multiplying the term $P[R(y) > D(\tau,y), y > x] / P[R(y) > D(\tau, y), y > x]$ and combining the first term on the right-hand side of Eq. (3) with the numerator, one obtains

$$\begin{aligned} f_{\Delta T\Delta\alpha}(\tau, x)\,\mathrm{d}r\,\mathrm{d}x &= P[R(y) > D(\tau, y),\ \forall y \neq x] \\ &\times P[D(\tau, x) < R(x) \le D(\tau + \Delta\tau, x)] \\ &\times \frac{P[R(y) > D(\tau + \Delta\tau, y),\ \forall y > x]}{P[R(y) > D(\tau, y),\ \forall y > x]} \end{aligned} \tag{4}$$

Each term on the right-hand side of Eq. (4) will be derived using Eq. (2) as follows:

Since $P[D(\tau, x) < R(x) \le D(\tau + \Delta\tau, x)] = P[R(x) \le D(\tau + \Delta\tau, x)] - P[R(x) \le D(\tau, x)] = F_{R(x)}[D(\tau + \Delta\tau, x)] - F_{R(x)}[D(\tau, x)]$, application of Eq. (2) yields

$$P[D(\tau, x) < R(x) \le D(\tau + \Delta\tau, x)]$$

$$= \frac{1}{\nu}\left[\frac{D(\tau+\Delta\tau, x) - \epsilon}{\beta}\right]^{\alpha} \Delta x - \frac{1}{\nu}\left[\frac{D(\tau, x) - \epsilon}{\beta}\right]^{\alpha} (\Delta x) \tag{5}$$

Further, using Eq. (2) and the fact that the strength of each element is statistically independent, one obtains

$$q = P[R(y) > D(\tau, y),\ \forall y \neq x]$$

$$= \prod_{j-1}^{\infty}\left\{1 - \frac{1}{\nu}\left[\frac{D(\tau, y_j) - \epsilon}{\beta}\right]^{\alpha} \Delta y_j\right\} \tag{6}$$

Taking the logarithm of Eq. (6), one obtains the following:

$$\ell n\, q = \sum_{j-1}^{\infty} \ell n\left\{1 - \frac{1}{\nu}\left[\frac{D(\tau, y_j) - \epsilon}{\beta}\right]^{\alpha} \Delta y_j\right\}$$

$$= -\sum_{j-1}^{\infty}\left\{\frac{1}{\nu}\left[\frac{D(\tau, y_j) - \epsilon}{\beta}\right]^{\alpha} \Delta y_j\right\}$$

in which the property that $\ell n(1-\epsilon) = -\epsilon$ for $\epsilon \to 0$ and the fact that Δy_j is very small have been used. Thus, q can be obtained from the preceding expression as

$$q = \exp\left\{-\sum_{j-1}^{\infty} \frac{1}{\nu}\left[\frac{D(\tau, y_j) - \epsilon}{\beta}\right]^{\alpha} \Delta y_j\right\}$$

$$= \exp\left\{-\int_0^W \frac{1}{\nu}\left[\frac{D(\tau, y_j) - \epsilon}{\beta}\right]^{\alpha} \mathrm{d}y\right\} \tag{7}$$

in which W is the failure process zone size ahead of the crack tip. It is assumed that the crack increment Δa is always smaller than the size of the failure process zone W. Such an assumption is quite reasonable, because the damage outside the failure process zone is negligible. The validity of these assumptions will be discussed later.

In the same manner as Eq. (7) is derived, one can show that

$$P[R(y) > D(\tau + \Delta\tau, y),\ \forall y > x]$$

$$= \exp\left\{-\int_x^W \frac{1}{\nu}\left[\frac{D(\tau+\Delta\tau, y_j) - \epsilon}{\beta}\right]^{\alpha} \mathrm{d}y\right\} \tag{8}$$

and

$$P\,[R(y) > D(\tau, y),\ \forall y > x]$$

$$= \exp\left\{-\int_x^W \frac{1}{v}\left[\frac{D(\tau, y_j) - \epsilon}{\beta}\right]^\alpha dy\right\} \tag{9}$$

Substituting Eqs. (5) and (7–9) into Eq. (4), dividing both sides by $\Delta\tau\Delta x$, and taking the limit as $\Delta\tau \to 0$, one obtains the joint density function of ΔT and Δa as follows:

$$f_{\Delta T \Delta \alpha}(\tau, x) = \exp\left\{-\int_0^W \frac{1}{v}\left[\frac{D(\tau, y_j) - \epsilon}{\beta}\right]^\alpha dy\right\}$$

$$\times \frac{d}{d\tau}\left\{\frac{1}{v}\left[\frac{D(\tau, x) - \epsilon}{\beta}\right]^\alpha x\right\} \tag{10}$$

It is noticed that Eq. (8) is identical to Eq. (9) as $\Delta\tau \to 0$, and Eq. (5) becomes the derivative of $v^{-1}\{[D(\tau, y) - \epsilon]/\beta\}^\alpha$ after dividing by $\Delta\tau$.

In reality, the distribution of the strength can be fitted by the two-parameter Weibull distribution, i.e., $\epsilon = 0$. In what follows, this approximation will be used. With the joint density function of Δa and ΔT derived earlier, the statistics of the crack growth rate $\dot{a}(t)$, given by Eq. (1), can be determined. In particular, the average crack growth rate and the coefficient of variation will be derived in the following.

The average crack growth rate, denoted by $E[\dot{a}(t)]$, is obtained from Eq. (1) as

$$E[\dot{a}(t)] = E[\Delta a/\Delta T] = \int_0^\infty \int_0^W \frac{x}{\tau} f_{\Delta T \Delta \alpha}(\tau, x)\, dx\, d\tau$$

$$= \int_0^\infty \frac{d\tau}{\tau} \int_0^W x \exp\left\{-\int_0^W \left[\frac{D(\tau, y)}{\beta v^{1/\alpha}}\right]^\alpha dy\right\}$$

$$\times \frac{d}{d\tau}\left\{\left[\frac{D(\tau, x)}{\beta v^{1/\alpha}}\right]^\alpha\right\} dx \tag{11}$$

where $E[\ ]$ denotes the ensemble average of the bracketed quantity and Eq. (10) has been used.

It is observed from Eqs. (10) and (11) that the average crack growth rate depends on the failure process zone size W, the strength of material elements defined by α and β, and the damage function $D(\tau, x)$ for an element at a distance x from the crack tip during the time period τ. Note that $\tau \le \Delta T$ is the time period within the successive events of the crack increment Δa.

The failure process zone can be expressed in terms of the mode I stress intensity factor K_I as[7]

$$W = \frac{\pi}{8}\left[\frac{K_I}{\sigma_f}\right]^2 \tag{12}$$

in which σ_f is the average failure stress in the failure process zone, i.e., $\sigma_f = \beta\Gamma(1 - \alpha^{-1})$. A possible damage function is assumed to have the following form:

$$D(\tau, y) = b\tau t_1^{-1} e^{-gy/W} \tag{13}$$

in which b and g are constants and t_1 is a unit time. Note that the dimension of b is identical to that of β, whereas g is a nondimensional quantity. The damage function given by Eq. (13) seems to be reasonable based on the results of nondestructive evaluation.[21,22]

Substituting Eqs. (10), (12), and (13) into Eq. (11) and carrying out the integration, one obtains the average crack growth rate as follows:

$$E[\dot{a}] = K_I^{2(1+\gamma)} C^{1+\gamma} (b t_1^{-1}/\beta v^\gamma)(\alpha g)^{-(1+\gamma)} [1 - e^{-\alpha g}]^{\gamma - 1}$$

$$\times [1 - (1 + \alpha g) e^{-\alpha g}]\, \Gamma(1 - \gamma) \tag{14}$$

in which $\Gamma(\ )$ is the gamma function and

$$C = \pi/\{8[\beta\Gamma(1-\gamma)]^2\}$$

$$\gamma = 1/\alpha$$

where the relation $\sigma_f = \beta\Gamma(1 - \gamma)$ has been used.

The mean square of the crack growth rate is given by

$$E[\dot{a}^2] = \int_0^\infty \int_0^W \left(\frac{x}{\tau}\right)^2 \exp\left\{-\int_0^W \left[\frac{D(\tau, y)}{\beta v^\gamma}\right]^\alpha dy\right\}$$

$$\times \frac{d}{d\tau}\left\{\left[\frac{D(\tau, x)}{\beta v^\gamma}\right]^\alpha\right\} d\tau\, dx \tag{15}$$

The standard deviation of the crack growth rate, denoted by $\sigma_{\dot{a}}$, and the coefficient of variation of crack growth rate, denoted by $V_{\dot{a}}$, are given by

$$\sigma_{\dot{a}} = \{E[\dot{a}^2] - [E(\dot{a})]^2\}^{1/2} \tag{16}$$

and

$$V_{\dot{a}} = \alpha_{\dot{a}}/E[\dot{a}] \tag{17}$$

Substituting Eqs. (12) and (13) into Eq. (15) and carrying out the integration, we obtain $E[\dot{a}^2]$. Then, from Eq. (16) and (17), the coefficient of variation of the crack growth rate is obtained as

$$V_{\dot{a}} = (F_1 - F_2)/F_3 \tag{18}$$

in which

$$F_1 = 2\Gamma(1 - 2\gamma)(1 - e^{-\alpha g})^{2(\gamma-1)}\left[1 - \left(1 + \alpha g + \frac{\alpha^2 g^2}{2}\right)e^{-\alpha g}\right] \tag{19}$$

$$F_2 = [\Gamma(1-\gamma)]^2 (1 - e^{-\alpha g})^{2(\gamma-1)} [1 - (1 + \alpha g) e^{-\alpha g}]^2 \tag{20}$$

$$F_3 = \Gamma(1-\gamma)(1 - e^{-\alpha g})^{(\gamma-1)} [1 - (1 + \alpha g) e^{-\alpha g}] \tag{21}$$

Discussion

In developing the crack growth model, it is assumed that the crack increment Δa is smaller or equal to the size of the failure process zone and that the material outside the failure process zone is not damaged. These assumptions are consistent with experimental findings if the applied strain is not large enough to cause a large amount of damage ahead of the failure process zone. According to the experimental data, it is found that, during the stable crack growth stage, the crack tip begins to blunt as soon as it is opened. Continued blunting under increasing applied strain initiates a damage zone, or failure process zone, in the immediate vicinity of the crack tip. When the local strain inside the failure process zone reaches a critical value, small voids are formed in the failure process zone. Because of the random nature of the microstructure of the material, the first void is not necessarily formed in the immediate neighborhood of the crack tip. The formation of large numbers of voids in the failure process zone leads to the formation of a large number of strands, which separate the voids, in the failure

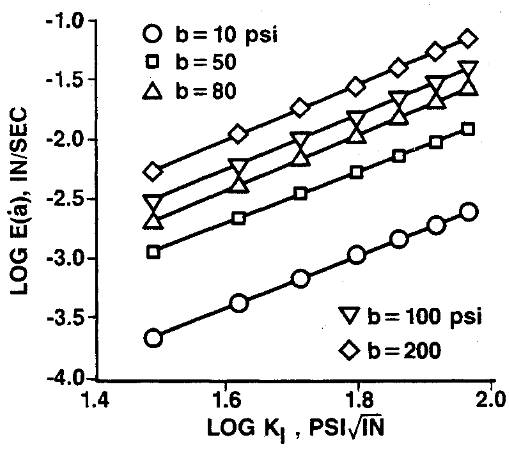

Fig. 2 Crack growth rate vs stress intensity factor; $g = 0.5$, $\alpha = 20$, $\beta = 108$ psi.

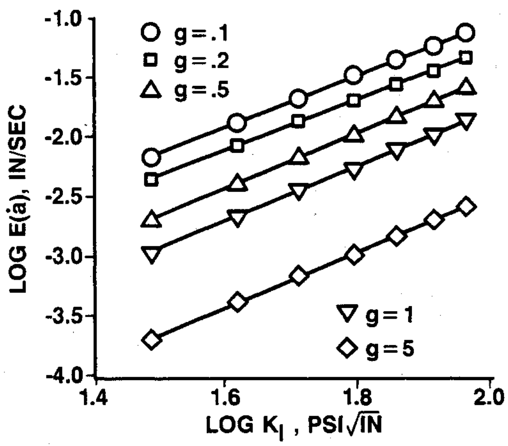

Fig. 3 Crack growth rate vs stress intensity factor; $b = 80$ psi, $\alpha = 20$, $\beta = 108$ psi.

process zone. As the applied strain is increased with time, the coalescence of voids occurs by the fracturing of strands between the voids under an essentially uniaxial loading condition. Finally, the failure of the material inside the failure process zone leads to crack growth to a distance which is equal to the length of the failure process zone. These damage initiation and evolution processes, as well as crack growth, are time-dependent processes that are responsible for the time-dependent crack growth behavior.

Based on these experimental findings, it is reasonable to assume that the incremental crack length Δa cannot be larger than the failure process zone size. Experimental results[22] also reveal that the highly strained region is restricted to a relatively small zone at the crack tip. This highly strained zone can be considered as the failure process zone. The magnitude of the strain outside the failure process zone is approximately equal to the magnitude of the applied strain. If the magnitude of the applied strain is smaller than a threshold value, the material behaves linearly. Since the mechanical response of a composite solid propellant is closely related to the damage state in the material, the initial linear portion of the stress-strain curve is associated with the stretching of undamaged material, with the filler particles bonded to the binder. As the external load is continuously increased to a certain critical damage state, the rigidity of the material is thereby reduced. This critical damage state usually coincides with the transition from linear response to nonlinear behavior. In other words, the strain corresponding to the critical damage state defines the threshold value of the strain below which the intensity of the damage is negligible. Based on this discussion, it is reasonable to assume that if the applied strain is small, the material outside the failure process zone is not damaged. However, if the applied strain is significantly higher than the threshold strain, the material outside the failure process zone becomes highly nonlinear due to high-intensity damage developed in the material itself. Under this condition, not only is the material damaged outside the failure process zone, but also the damage gradient inside the failure process zone is not constant. Consequently, the basic assumptions used to develop the crack growth model are violated and the developed crack growth model cannot be used to predict the crack growth behavior.

The assumption that the strength of each small element in a material volume is statistically independent, as originally proposed by Weibull,[24] has been shown to be quite reasonable, and it has been used extensively in the literature.[15,16] When the width of each element ahead of the crack tip Δy is changed to dy, the strength $R(y)$ becomes a white noise process in the space coordinate y. Although one may introduce a correlation function for the strength $R(y)$, two difficulties arise: 1) it is not easy to determine the correlation distance for a particular material, such as the composite solid propellant, and 2) the analytical closed-form solutions for the first two statistical moments of the crack growth rate are not tractable. Because of these difficulties, the well-known Weibull assumption is used herein as a first-order approximation to establish a stochastic crack growth model.

The effect of damage intensity, in terms of b and g, and material strength, in terms of α and β, are shown in Figs. 2–5. It is observed in Fig. 2 and Eq. (14) that the average crack growth rate is directly proportional to b, and a one order of magnitude increase in b results in a one order of magnitude increase in the crack growth rate. This indicates that the crack growth rate is closely related to the damage intensity near the crack tip. The effects of g and α on the crack growth rate are not easily determined by examining Eq.

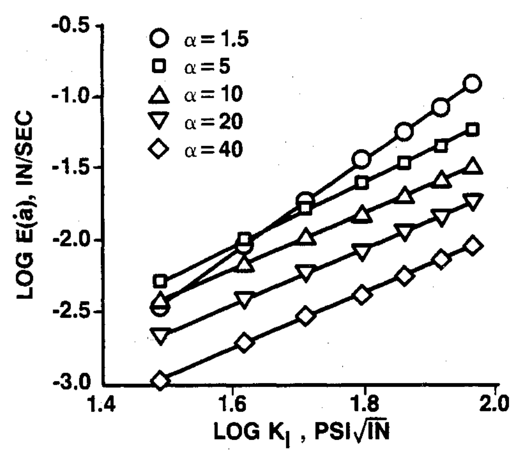

Fig. 4 Crack growth rate vs stress intensity factor; $b = 80$ psi, $g = 0.5$, $\beta = 108$ psi.

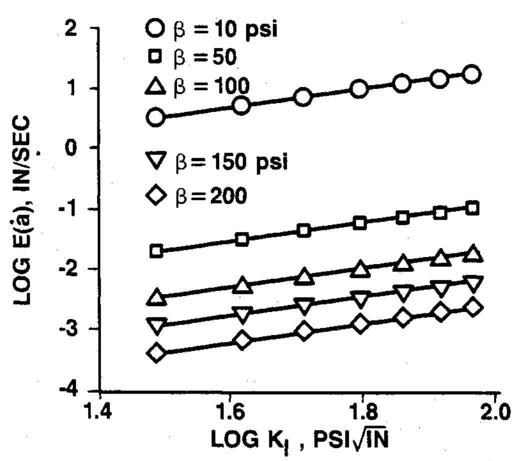

Fig. 5 Crack growth rate vs stress intensity factor; $b = 80$ psi, $g = 0.5$, $\alpha = 20$.

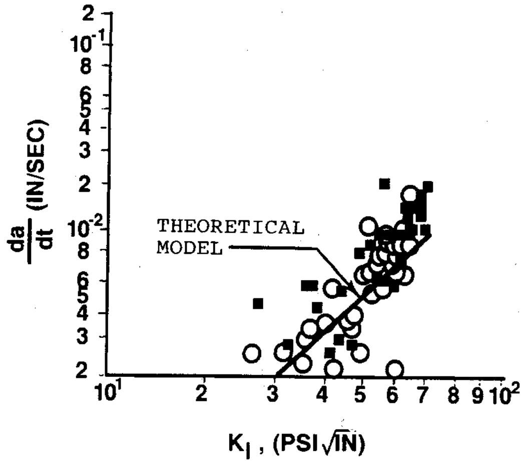

Fig. 6 Crack growth rate vs stress intensity factor; experimental data (Fig. 4 in Ref. 4).

(14) because of the complex functional dependence of the crack growth rate on g and α. However, from Fig. 3, it is clearly indicated that when g increases by one order of magnitude, the crack growth rate decreases approximately by one order of magnitude. For a given value of b, the increase in the crack growth rate with decreasing slope of the damage curve, or g, is due to the increase in the probability of having a material element whose strength is lower than the damage intensity of the material in the damage process zone. Also, from Fig. 3, we notice that the curves in the figure are parallel. As expected, the exponent in Eq. (14) is a function of α only. The effects of α on crack growth rate and the slope of the crack growth curve are shown in Fig. 4. As shown by Fig. 4, when $\alpha \geq 5$, the slope of the crack growth curve is relatively insensitive to the value of α. It is interesting to note that, when $\alpha < 5$, the slope of the crack growth curve increases with decreasing α. Note that α reflects the variability of the material strength, i.e., the smaller the α value, the larger the dispersion of the material strength. Figure 4 indicates that the crack growth rate increases as the variability of the material strength increases. One way of increasing the crack growth resistance is to reduce the variability of the strength of the composite solid propellants. Equation (14) indicates that the crack growth rate is inversely proportional to β. The degree of the effect of β on the crack growth rate is shown in Fig. 5. This figure also shows that when β increases by one order of magnitude, the crack growth rate decreases by approximately three orders of magnitude. Based on the results of these analyses, it is concluded that, among the parameters in the crack growth model, it is the characteristic value of the strength of the material β that has the greatest effect on the crack growth rate. This conclusion provides important guidance to the propellant formulation for increasing the crack growth resistance of the material.

It should be mentioned that the average crack growth rate, $E[\dot{a}]$, is always positive. Since $\Gamma(1-\gamma) = \Gamma(1-\alpha^{-1})$ becomes negative when $\alpha \leq 1$, it follows from Eq. (14) that γ should be smaller than 1.0 or $\alpha > 1$. When $\alpha = 1$, the distribution function of the strength becomes an exponential distribution and the coefficient of variation (dispersion) becomes 100%, a rare situation not expected from the experimental test results. Therefore, α is between 1 and ∞. Again referring to Eq. (14), it can be seen that the average crack growth rate $E[\dot{a}]$ is a power function of the stress intensity factor K_I, with the exponent being equal to $2(1 + \alpha^{-1})$. In other words, the slope of the log $E[\dot{a}]$ vs log K_I curve is $2(1 + \alpha^{-1})$ and lies between 2.0 (when $\alpha = \infty$) and 4.0 (when $\alpha = 1$). It is interesting to note that the form of the developed crack growth model, which is based on probabilistic mechanics, is identical to one developed by Schapery[7] based on linear viscoelastic fracture mechanics. Furthermore, the power law which relates the crack growth rate to the stress intensity factor has been supported by experimental data obtained from crack propagation tests of solid propellants.

Crack propagation test data, obtained from Ref. 4, is shown in Fig. 6. In addition, the predicted crack growth curve, based on the developed crack growth model, for $\alpha = 20.0$, $\beta = 108$ psi, $g = 0.5$, $b = 80$ psi, is also shown in Fig. 6. From that same figure, it can be seen that the theoretical analysis results and the experimental test data are in agreement. However, crack propagation test data of a different composite solid propellant indicate that the slope of the crack growth rate vs the stress intensity factor curve is equal to 6.0, which is outside the range of 2–4 predicted by the developed crack growth model. Thus, additional research efforts are needed to modify the developed crack growth model in order to expand the range of the slopes. This can be accomplished as indicated by our preliminary investigation by assuming that the damage function $D(\tau, y)$ is a function of the stress intensity factor K_I. Further results in this regard will be reported elsewhere.

Conclusions

In this study, a probabilistic crack growth model is developed. The developed crack growth model yields a power law relationship between the average crack growth rate and the mode I stress intensity factor. In other words, the average crack growth rate $E[\dot{a}]$ is a power function of the stress intensity factor K_I with the exponent equal to $2(1 + \alpha^{-1})$. The value of the exponent lies between 2.0 (when $\alpha = 1$) and 4.0 (when $a = \infty$). This range of theoretical values for the exponent has been supported by experimental data obtained from crack propagation tests conducted on a number of solid propellants. However, the developed model needs to be refined to extend the range of exponent values so that the model can be used to predict crack growth behavior of various solid propellants. In addition to predicting crack growth behavior, the developed crack growth model serves to guide the propellant formulation for increasing the crack growth resistance of the material.

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References

[1]Beckwith, S. W., and Wang, D. T., "Crack Propagation in Double-Base Propellants," AIAA Paper 78-170, Jan. 1978.

[2]Liu, C. T., "Variability in Crack Growth in a Composite Propellant," AIAA Paper 83-1015, May 1983.

[3]Liu, C. T., "Crack Growth Behavior in a Composite Propellant with Strain Gradients," AIAA Paper 84-1293, June 1984.

[4]Liu, C. T., "Crack Growth Behavior in a Composite Propellant with Strain Gradients—Part II," *Journal of Spacecraft and Rockets*, Vol. 27, No. 6, 1990, pp. 647–652.

[5]Liu, C. T., "Crack Propagation in a Composite Solid Propellant," *Proceedings of the Society of Experimental Mechanics*, Spring Conference, Bethel, CT, June 1990, pp. 614–620.

[6]Knauss, W. G., "Delayed Failure—The Griffith Problem for Linearly Viscoelastic Materials," *International Journal of Fracture Mechanics*, Vol. 6, March 1970, pp. 7–20.

[7]Schapery, R. A., "On a Theory of Crack Growth in Viscoelastic Media," Texas A & M Univ., MM 2765-73-1, College Station, TX, March 1973.

[8]Kozin, F., and Bogdanoff, J. L., "On the Probabilistic Modeling of Fatigue Crack Growth," *Journal of Engineering Fracture Mechanics*, Vol. 20, No. 3, 1984, pp. 623–632.

[9]Bogdanoff, J. L., and Kozin, F., "Probabilistic Models of Fatigue Crack Growth—II," *Journal of Engineering Fracture Mechanics*, Vol. 20, No. 2, 1984, pp. 255–270.

[10]Yang, J. N., Salivar, G., and Schwartz, B., "Statistical Modeling of Fatigue Crack Growth in a Nickel-Base Super-Alloy," *Journal of Engineering Fracture Mechanics*, Vol. 18, 1983, pp. 257–270.

[11]Lin, Y. K., and Yang, J. N., "On Statistical Moments of Fatigue Crack Propagation," *Journal of Engineering Fracture Mechanics*, Vol. 18, 1983, pp. 243–256.

[12]Lin, Y. K., and Yang, J. N., "A Stochastic Theory of Fatigue Crack Propagation," *AIAA Journal*, Vol. 23, No. 1, 1985, pp. 117–124.

[13]Yang, J. N., and Donath, R. C., "Statistical Fatigue Crack Propagation in Fastener Holes Under Spectrum Loading," *Journal of Aircraft*, Vol. 20, No. 6, 1983, pp. 1028–1032.

[14]Yang, J. N., and Chen, S., "Fatigue Reliability of Gas Turbine Engine Components Under Schedule Inspection Maintenance," *Journal of Aircraft,* Vol. 22, No. 5, 1985, pp. 415–422.

[15]Eggwertz, S., and Lin, N. C. (eds.), *Probabilistic Methods in Mechanics of Solids and Structures,* Springer-Verlag, New York, 1984.

[16]Provan, J. W. (ed.), *Probabilistic Fracture Mechanics and Reliability,* Martinus Nijhoff, Boston, MA, 1987.

[17]Yang, J. N., Manning, S. D., Hsi, W. H., and Rudd, J. L., "Stochastic Crack Growth Models for Applications to Aircraft Structures," *Probabilistic Fracture Mechanics,* edited by J.W. Provan, Martinus Nijhoff, Boston, MA, 1987, Chap. 4, pp. 171–211.

[18]Yang, J. N., and Manning, S. D., "Stochastic Crack Growth Analysis Methodologies for Metallic Structures," *Journal of Engineering Fracture Mechanics,* Vol. 37, No. 5, 1990, pp. 105–1124.

[19]Yang, J. N., and Donath, R. C., "Statistical Crack Growth of a Superalloy Under Sustained Loads," *Journal of Engineering Materials and Technology,* Vol. 106, Jan. 1984, pp. 79–83.

[20]Oh, K. P., "A Weakest-Link Model for the Prediction of Fatigue Crack Growth Rate," *Journal of Engineering Materials and Technology,* Vol. 100, April 1978, pp. 170–174.

[21]Liu, C. T., "Nondestructive Evaluation of Near Tip Damage Fields in a Composite Solid Propellant," *Proceedings of the International Conference on Computational Engineering Science,* Atlanta, GA, April 1988.

[22]Liu, C. T., "Measurement of Strain Field Near the Tip of a Crack in a Composite Solid Propellant," *Proceedings of the Society for Experimental Mechanics 1989 Spring Conference,* May 1989, pp. 491–496.

[23]Liu, C. T., "Crack Propagation in an HTPB Solid Propellant," *1980 JANNAF Structures and Mechanical Behavior Subcommittee Meeting,* Vol. I, CPIA Pub. 311, March 1980, pp. 193–205.

[24]Weibull, W., "A Statistical Theory of Strength of Materials," *Proceedings of the Royal Swedish Institute of Engineering Research,* Stockholm, Sweden, No. 151, 1939.

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